-"The importance of drill on components [such as math facts] is that the drilled material may become sufficiently over-learned to free up cognitive resources and attention. These cognitive resources may then be allocated to other aspects of performance, such as more complex operations like carrying and borrowing, and to self-monitoring and control (Goldman & Pellegrino, 1986, p.134)"

-"For students to be able to recall facts quickly in more complex computational problems, research tells us the students must know their math facts at an acceptable level of 'automaticity.' Therefore, teachers...must be prepared to supplement by providing more practice, as well as by establishing rate criteria that students must achieve. (Stein et al., 1997, p. 93)."

Math Fluency

The experts agree that the ability to recall basic math facts fluently is necessary for students to attain higher-order math skills.

Educators and cognitive scientists agree that the ability to recall basic math facts fluently is necessary for students to attain higher-order math skills. Grover Whitehurst, the Director of the Institute for Educational Sciences (IES), noted this research during the launch of the federal Math Summit in 2003: “Cognitive psychologists have discovered that humans have fixed limits on the attention and memory that can be used to solve problems. One way around these limits is to have certain components of a task become so routine and over-learned that they become automatic.” Whitehurst, 2003)

The implication for mathematics is that some of the sub-processes, particularly basic facts, need to be developed to the point that they are done automatically. If this fluent retrieval does not develop then the development of higher-order mathematics skills — such as multiple-digit addition and subtraction, long division, and fractions — may be severely impaired. Indeed, studies have found that lack of math fact retrieval can impede participation in math class discussions, successful mathematics problem-solving, and even the development of everyday life skills. And rapid math-fact retrieval has been shown to be a strong predictor of performance on mathematics achievement tests.

If a student constantly has to compute the answers to basic facts, less of that student's thinking capacity can be devoted to higher level concepts than a student who can effortlessly recall the answers to basic facts. For example, a child who is performing multiple-digit division must monitor constantly where he is in that procedure. If the child must use primitive counting strategies to subtract or multiply during the division process, the attention and memory resources devoted to these procedures reduce the student's ability to monitor and attend to the larger division problem. The result is that the student often fails to grasp the concepts involved in multiple-digit division.

Recent research in cognitive science, using functional magnetic resonance imaging (fMRI), has revealed the actual shift in brain activation patterns as untrained math facts are learned (Delazer et al., 2003). As predicted by Dehaene (1997, 1999, 2003), instruction and practice cause math fact processing to move from a quantitative area of the brain to one related to automatic retrieval. Delazer and her colleagues suggest that this shift aids the solving of complex computations that require "the selection of an appropriate resolution algorithm, retrieval of intermediate results, storage and updating in working memory" by substituting some of the intermediate steps with automatic retrieval (Delazer et al., 2004).

The research cited above highlights the importance of math fact fluency; however, the computation capabilities of American students appear to be falling. Tom Loveless of the Brookings Institute has reviewed responses to select items on the National Assessment of Educational Progress (NAEP) and concluded that performance on basic arithmetic facts declined in the 1990s (Loveless, 2003). Clearly, students need help to develop rapid, effortless, and errorless recall of basic math facts.

Mathematical Knowledge

Mathematical knowledge of basic facts can be classified into two categories. The first category, called *declarative knowledge*, can be conceptualized as an interrelated network of relationships containing basic problems and their answers, such as 4+7=11 or 11-4=7. The facts stored in this network have different "strengths" that determine how long it takes to retrieve an answer. The stronger the relationship, the more rapid and effortless is the retrieval process. For example, if the fact 2+3=5 has greater associative strength than the fact 7+5=12, it will take less time to retrieve the answer 5 to the first of these two problems (Pellegrino & Goldman, 1987).

Ideally, all the facts stored in this network are retrieved from memory quickly, effortlessly, and without error. However, this is often not the case with many students, particularly those with learning problems. These students, for a variety of reasons, have not established a declarative knowledge network, and instead of retrieving facts from memory, they rely on a second category of mathematics knowledge, called procedural knowledge.

Procedural knowledge refers to methods that can be used to derive answers for problems lacking pre-stored answers. For example, in the problem 6+8, a student might use a common "counting-on" strategy in which the larger of the two addends (8) is stated and the student increments the smaller addend on his or her fingers while saying 9, 10, 11, 12, 13, 14. Although correct answers can be obtained using procedural knowledge, these procedures are effortful, slow, error-prone, and they appear to interfere with learning and understanding higher-order concepts.

Underlying both declarative and procedural knowledge in mathematics is a type of understanding typically called number sense. While several definitions of number sense can be found (see, for instance, NCTM Standards 2004 or Case 1998), academics generally agree that it involves an awareness of number names, values, and relationships. Children with number sense recognize the relative differences in number quantity and how those differences can be represented. Number sense gives meaning both to an automatic math fact and to a computational procedure. Gersten and Chard roughly compare the importance of number sense in computation to the need for phonemic awareness in reading (Gersten & Chard, 1999). Both are critical building blocks. Garnett describes a typical hierarchy of procedures, or strategies, that rests upon number sense and leads eventually to automatic recall (Garnett, 1992). All elements—number sense, procedural knowledge, and declarative knowledge — must be developed together to achieve full math fact fluency.

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