MAA

MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

14th Annual

AMC 10 A

American Mathematics Contest 10 A

Tuesday, February 5, 2013

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 31st annual American Invitational Mathematics Examination (AIME) on Thursday, March 14, 2013 or Wednesday, April 3, 2013. More details about the AIME and other information are on the back page of this test booklet.

2013

AMC 10 A

DO NOT OPEN UNTIL TUESDAY, February 5, 2013

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 5, 2013. Nothing is needed from inside this package until February 5.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, email, internet or media of any type is a violation of the competition rules.*

The American Mathematics Competitions

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1. A taxi ride costs \$1.50 plus \$0.25 per mile traveled. How much does a 5-mile taxi ride cost?

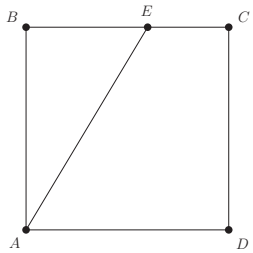
   (A) \$2.25 **(B)** \$2.50 **(C)** \$2.75 **(D)** \$3.00 **(E)** \$3.25

2. Alice is making a batch of cookies and needs $2\frac{1}{2}$ cups of sugar. Unfortunately, her measuring cup holds only $\frac{1}{4}$ cup of sugar. How many times must she fill that cup to get the correct amount of sugar?

   (A) 8 **(B)** 10 **(C)** 12 **(D)** 16 **(E)** 20

3. Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8



4. A softball team played ten games, scoring 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 runs. They lost by one run in exactly five games. In each of their other games, they scored twice as many runs as their opponent. How many total runs did their opponents score?

   (A) 35 **(B)** 40 **(C)** 45 **(D)** 50 **(E)** 55

5. Tom, Dorothy, and Sammy went on a vacation and agreed to split the costs evenly. During their trip Tom paid \$105, Dorothy paid \$125, and Sammy paid \$175. In order to share the costs equally, Tom gave Sammy t dollars, and Dorothy gave Sammy d dollars. What is $t - d$?

   (A) 15 **(B)** 20 **(C)** 25 **(D)** 30 **(E)** 35

6. Joey and his five brothers are ages 3, 5, 7, 9, 11, and 13. One afternoon two of his brothers whose ages sum to 16 went to the movies, two brothers younger than 10 went to play baseball, and Joey and the 5-year-old stayed home. How old is Joey?

   (A) 3 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 13

7. A student must choose a program of four courses from a menu of courses consisting of English, Algebra, Geometry, History, Art, and Latin. This program must contain English and at least one mathematics course. In how many ways can this program be chosen?

   (A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 16

8. What is the value of
$$\frac{2^{2014}+2^{2012}}{2^{2014}-2^{2012}}?$$

   (A) -1 **(B)** 1 **(C)** $\frac{5}{3}$ **(D)** 2013 **(E)** 2^{4024}

9. In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on 20% of her three-point shots and 30% of her two-point shots. Shenille attempted 30 shots. How many points did she score?

   (A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

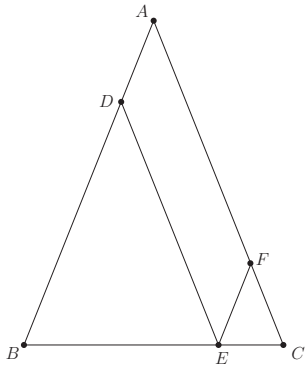
10. A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

    (A) 15 **(B)** 30 **(C)** 40 **(D)** 60 **(E)** 70

11. A student council must select a two-person welcoming committee and a three-person planning committee from among its members. There are exactly 10 ways to select a two-person team for the welcoming committee. It is possible for students to serve on both committees. In how many different ways can a three-person planning committee be selected?

    (A) 10 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 25

12. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$?



(A) 48 **(B)** 52 **(C)** 56 **(D)** 60 **(E)** 72

13. How many three-digit numbers are not divisible by 5, have digits that sum to less than 20, and have the first digit equal to the third digit?

(A) 52 **(B)** 60 **(C)** 66 **(D)** 68 **(E)** 70

14. A solid cube of side length 1 is removed from each corner of a solid cube of side length 3. How many edges does the remaining solid have?

(A) 36 **(B)** 60 **(C)** 72 **(D)** 84 **(E)** 108

15. Two sides of a triangle have lengths 10 and 15. The length of the altitude to the third side is the average of the lengths of the altitudes to the two given sides. How long is the third side?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 18

16. A triangle with vertices $(6, 5)$, $(8, -3)$, and $(9, 1)$ is reflected about the line $x = 8$ to create a second triangle. What is the area of the union of the two triangles?

(A) 9 **(B)** $\frac{28}{3}$ **(C)** 10 **(D)** $\frac{31}{3}$ **(E)** $\frac{32}{3}$

17. Daphne is visited periodically by her three best friends: Alice, Beatrix, and Claire. Alice visits every third day, Beatrix visits every fourth day, and Claire visits every fifth day. All three friends visited Daphne yesterday. How many days of the next 365-day period will exactly two friends visit her?

(A) 48 **(B)** 54 **(C)** 60 **(D)** 66 **(E)** 72

18. Let points $A = (0,0)$, $B = (1,2)$, $C = (3,3)$, and $D = (4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $(\frac{p}{q}, \frac{r}{s})$, where these fractions are in lowest terms. What is $p+q+r+s$?

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

19. In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b representation of 2013 end in the digit 3?

(A) 6 **(B)** 9 **(C)** 13 **(D)** 16 **(E)** 18

20. A unit square is rotated 45° about its center. What is the area of the region swept out by the interior of the square?

(A) $1 - \frac{\sqrt{2}}{2} + \frac{\pi}{4}$ **(B)** $\frac{1}{2} + \frac{\pi}{4}$ **(C)** $2 - \sqrt{2} + \frac{\pi}{4}$

(D) $\frac{\sqrt{2}}{2} + \frac{\pi}{4}$ **(E)** $1 + \frac{\sqrt{2}}{4} + \frac{\pi}{8}$

21. A group of 12 pirates agree to divide a treasure chest of gold coins among themselves as follows. The k^{th} pirate to take a share takes $\frac{k}{12}$ of the coins that remain in the chest. The number of coins initially in the chest is the smallest number for which this arrangement will allow each pirate to receive a positive whole number of coins. How many coins does the 12^{th} pirate receive?

(A) 720 **(B)** 1296 **(C)** 1728 **(D)** 1925 **(E)** 3850

22. Six spheres of radius 1 are positioned so that their centers are at the vertices of a regular hexagon of side length 2. The six spheres are internally tangent to a larger sphere whose center is the center of the hexagon. An eighth sphere is externally tangent to the six smaller spheres and internally tangent to the larger sphere. What is the radius of this eighth sphere?

(A) $\sqrt{2}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{5}{3}$ **(D)** $\sqrt{3}$ **(E)** 2

23. In $\triangle ABC$, $AB = 86$, and $AC = 97$. A circle with center A and radius AB intersects $\overline{BC}$ at points B and X. Moreover $\overline{BX}$ and $\overline{CX}$ have integer lengths. What is BC?

    (A) 11 **(B)** 28 **(C)** 33 **(D)** 61 **(E)** 72

24. Central High School is competing against Northern High School in a backgammon match. Each school has three players, and the contest rules require that each player play two games against each of the other school's players. The match takes place in six rounds, with three games played simultaneously in each round. In how many different ways can the match be scheduled?

    (A) 540 **(B)** 600 **(C)** 720 **(D)** 810 **(E)** 900

25. All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?

    (A) 49 **(B)** 65 **(C)** 70 **(D)** 96 **(E)** 128

American Mathematics Competitions

WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

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The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

Dr. Leroy Wenstrom

2013 AIME

The 31st annual AIME will be held on Thursday, March 14, with the alternate on Wednesday, April 3. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 42nd Annual USA Mathematical Olympiad (USAMO) on April 30-May 1, 2013. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: amc.maa.org